RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2023

SECOND YEAR [BATCH 2021-24]

Date : 25/05/2023 **PHYSICS (HONOURS)**

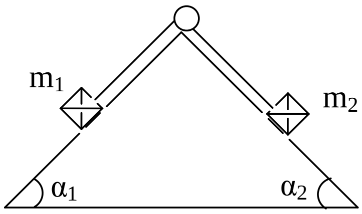
Time : 11 am – 1 pm **Paper : CC9** Full Marks : 50

Answer **any five** questions: [5×10]

1. a) What is meant by a virtual displacement. Explain the difference between actual and virtual displacements with appropriate examples.

   b) Explain the nature of the constraints for the following cases:
   (i) air molecules confined in a room,
   (ii) a particle moving on the surface of a sphere whose radius is a function of time such that $R(t) = R_0 + at$.
   (iii) a disc rolling on an inclined plane without slipping.

   c) Two masses m_1 and m_2 are connected by an in extensible string which passes own a smooth pully of negligible mass. The masses are placed on two inclined planes as shown as the figure. Use D'Alembert's principle of virtual work to show that $m_1 \sin\alpha_1 = m_2 \sin\alpha_2$. [3+3+4]



2. a) Show that Lagrange's equations are invariant under Galleian transformation.

   b) Two particles of masses m_1 and m_2 and position vectors $\vec{r}_1$ and $\vec{r}_2$ interact via a potential $V(|\vec{r}_1 - \vec{r}_2|)$. Write down the Lagrangian and hence obtain the corresponding equations of motion in terms of the centre of mass coordinate $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2)$ and the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$. Deduce the constant of motion, if any. [5+5]

3. a) State Hamilton's principle of least action.

   b) The Lagrangian of an an-harmonic oscillator is given by
   $$L(x, \dot{x}) = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta x \dot{x}^2.$$
   Obtain the Hamiltonian of the system and find the equation of motion.

   c) Prove that the transformation: $Q = q\tan p\,, P = \ln\sin p$ is canonical.

   d) Show that $\frac{du}{dt} = [u, H] + \frac{\partial u}{\partial t}$ (symbols have their usual meanings). [2+(2+2)+2+2]

4. a) For a system of n particles executing small oscillations under stable equilibrium deduce the set of Lagrange's equations of motion.

   b) The Lagrangian of a system is given by $L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$. Find the frequencies of normal modes.

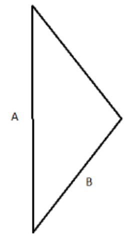
   c) Derive the Euler's equations for the motion of a rigid body with one point fixed in space under the action of a torque. [3+4+3]

5. a) State two basic postulates of relativity. Using the basic postulates derive the Lorentz transformation equations.

   b) An astronaut wants to reach a star which is 1 light year away from him. What should the the speed of his space ship so that he can reach there is one year.

c) Clock A is at rest in our frame of reference, and clock B is moving at speed 0.6 c relative to us. Just as clock B passes clock A, both clocks reach 12:00 midnight. (i) When clock reads 5:00, five hours later, what does clock B, read as observed in our frame? (ii) When clock B reads the time found in part (i), what does clock A read as observed in B's frame. [(2+4)+2+(1+1)]

6. a) In the 'paradox' of twins A and B, A stays home and B travels to a distant star, then turns around and comes home. In spacetime, their world lines are as shown in the adjacent figure. Using our sign convention,
(i) Are the spacetime intervals of A and B positive or negative?
(ii) Which has the larger magnitude?
(iii) Which has experienced the larger proper time?



b) Derive an expression for the relativistic Doppler effect. Show that under small velocity approximation it will reduce to the classical expression of Doppler effect. [(1+1+1)+(5+2)]

7. a) Using the work energy theorem, to show that the relativistic kinetic energy of a particle is $T=(m-m_0)c^2$ where the symbols have usual meanings. Here establish the energy momentum relation.

b) The total energy of a proton that has been accelerated in a synchrotron is 30 times its rest mass energy $m_p c^2$. In terms of m_p, find the proton's kinetic energy and the magnitude of its momentum. Also find its velocity.

c) Show that the energy and momentum transform as

$$p_x = \gamma\left(p'_x + \frac{E'v}{c^2}\right), p_y = p'_y, p_z = p'_z, E = \gamma(E' + vp'_x)$$

Hint: $\frac{1}{\sqrt{1-u^2/c^2}} = \frac{\gamma\left(1+u_x'v/c^2\right)}{\sqrt{1-u'^2/c^2}}$, where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. [(2+1)+(1+1+1)+4]

8. a) Show that the electric field components transforms as

$$E'_x = E_x, E'_y = \gamma(E_y - vB_z), E'_z = \gamma(E_z + vB_y)$$

By considering a charge particle q to be instantaneously at rest in S' frame and S' is moving with constant speed v along xx'-axis with respect to S.

b) Show that the magnetic field transforms as,

$$B'_x = B_x, B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

By considering the charge particle q moving with speed u' in the transverse direction in S' frame and S' is moving with constant speed v along xx'-axis with respect to S.

Hint: The forces transforms as

$$F'_x = \frac{F_x - (v/c^2)\vec{u}\cdot\vec{F}}{\left(1 - u_x v/c^2\right)}, F'_y = \frac{F_y}{\gamma\left(1 - u_x v/c^2\right)}, F'_z = \frac{F_z}{\gamma\left(1 - u_x v/c^2\right)}$$

[3+7]

——— × ———